Answers for Quiz 5

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1. 1. $1^3 = \frac{1^2(1+1)^2}{4}$
   2. $\frac{1^2(1+1)^2}{4} = \frac{4}{4} = 1 = 1^3$
   3. For an arbitrary $m \geq 1$, $1^3 + 2^3 + \cdots + m^3 = \frac{m^2(m+1)^2}{4}$
   4. $1^3 + 2^3 + \cdots + (m+1)^3 = \frac{(m+1)^2(m+2)^2}{4}$
   5.
   $$\begin{aligned} 1^3 + 2^3 + \cdots + m^3 + (m+1)^3 &= \frac{m^2(m+1)^2}{4} + (m+1)^2(m+1) \\ &= (m+1)^2\left(\frac{m^2}{4} + m + 1\right) \\ &= (m+1)^2\left(\frac{m^2+4m+4}{4}\right) \\ &= \frac{(m+1)^2(m+2)^2}{4} \end{aligned}$$
   6. Suppose there is a non-empty set S of positive integers for which the equality does not hold. By well ordering, S must have a least element, k. We proved $P(1)$, so $k \neq 1$. Therefore, there must be a positive integer $j \notin S$ such that $j+1 = k$. But the inductive step shows that $P(j) \to P(j+1)$, and therefore $k \notin S$. This produces a contradiction, so S must be empty and the equality must hold for all positive integers.

      Alternate answer: We showed $P(1)$, and by the inductive step $P(1) \to P(2)$, and $P(2) \to P(3)$, and so forth.

2. 1. 6
   2. 12
   3. 20
   4. $n^2 + n$

   Substitute your own formula for $n^2 + n$ in the answers below.

   1. $2 + 4 + 6 + \cdots + 2n = n^2 + n$
   2. For an arbitrary $k \geq 1$, $2 + 4 + \cdots + 2k = k^2 + k$
   3. $2 + 4 + \cdots + 2(k+1) = (k+1)^2 + (k+1)$