Answers to Quiz 2

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1. (a) (i) $p \wedge q$, where p means "2 is even" and q means "2 is prime"
   (ii) $\neg(p \vee q)$, where p means "9 is even" and q means "9 is prime"
   (iii) $r \to (p \wedge \neg q)$, where p means "8 is odd", q means "8 is prime", and r means "8 is a multiple of 4"

   (b) $|x - 2| \leq 2$ and $-2 \leq x \leq 2$
   Note that $\neg(p \to (q \vee r)) \equiv \neg p \wedge \neg(q \vee r) \equiv \neg p \wedge \neg q \wedge \neg r$.

2. (a) (i)

$$
\begin{aligned}
(p \wedge q) \to p &\equiv \neg(p \wedge q) \vee p && \to \text{Equivalence} \\
&\equiv \neg p \vee \neg q \vee p && \text{DeMorgan's Laws} \\
&\equiv \neg p \vee p \vee \neg q && \text{Commutative} \\
&\equiv \text{True} \vee q && \text{Inverse (or Negation)} \\
&\equiv \text{True} && \text{Domination}
\end{aligned}
$$

(ii)

$$
\begin{aligned}
\neg(p \to q) \leftrightarrow (p \wedge \neg q) &\equiv \neg(\neg p \vee q) \leftrightarrow (p \wedge \neg q) && \to \text{Equivalence} \\
&\equiv (p \wedge \neg q) \leftrightarrow (p \wedge \neg q) && \text{DeMorgan's Laws} \\
&\equiv ((p \wedge \neg q) \to (p \wedge \neg q)) \wedge ((p \wedge \neg q) \to (p \wedge \neg q)) && \leftrightarrow \text{Equivalence} \\
&\equiv (p \wedge \neg q) \to (p \wedge \neg q) && \text{Idempotence} \\
&\equiv \neg(p \wedge \neg q) \vee (p \wedge \neg q) && \to \text{Equivalence} \\
&\equiv \text{True} && \text{Inverse}
\end{aligned}
$$

(iii)

$$
\begin{aligned}
(\neg p \leftrightarrow q) \leftrightarrow (p \leftrightarrow \neg q) &\equiv ((\neg p \to q) \wedge (q \to \neg q)) \leftrightarrow ((p \to \neg q) \wedge (\neg q \to p)) && \leftrightarrow \text{Equivalence} \\
&\equiv ((\neg\neg p \vee q) \wedge (\neg q \vee \neg p)) \leftrightarrow ((\neg p \vee \neg q) \wedge (\neg\neg q \wedge p)) && \to \text{Equivalence} \\
&\equiv ((p \vee q) \wedge (\neg q \vee \neg p)) \leftrightarrow ((\neg p \vee \neg q) \wedge (q \vee p)) && \text{Double Negation} \\
&\equiv ((p \vee q) \wedge (\neg p \wedge \neg q)) \leftrightarrow ((p \vee q) \wedge (\neg p \vee \neg q)) && \text{Commutative} \\
&\equiv (((p \vee q) \wedge (\neg p \wedge \neg q)) \to ((p \vee q) \wedge (\neg p \wedge \neg q))) \wedge \\
&\qquad (((p \vee q) \wedge (\neg p \wedge \neg q)) \to ((p \vee q) \wedge (\neg p \wedge \neg q))) && \leftrightarrow \text{Equivalence} \\
&\equiv ((p \vee q) \wedge (\neg p \wedge \neg q)) \to ((p \vee q) \wedge (\neg p \wedge \neg q)) && \text{Idempotence} \\
&\equiv \neg((p \vee q) \wedge (\neg p \vee \neg q)) \vee ((p \vee q) \wedge (\neg p \vee \neg q)) && \to \text{Equivalence} \\
&\equiv \text{True} && \text{Inverse}
\end{aligned}
$$

(b) $\forall x(D(x) \to W(x))$

3. 1. False. $1/2 \notin \mathbb{N}$
   2. True. $1/2 \in \mathbb{Q}$
   3. True. Say there is a y_0 such that $\forall x P(x, y_0)$. Then, for any x_0, $\exists y P(x_0, y)$ because $P(x_0, y_0)$.
   4. False. $\forall x \exists y P(x, y)$ means that for an x_0 there is a y_0 such that $P(x_0, y_0)$, and for x_1 there is a y_1 such that $P(x_1, y_1)$, but there is not necessarily a y_i such that $\forall x P(x, y_i)$.