

Answers for Quiz 1

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1. (a) $A \cap (A \cup B) = A$ (by Absorption)
(b) (i) $3 \in \{3, 3\}; 3 \notin \{\{\{3\}\}, \{3\}\}; 3 \notin \{\{\{\{3\}\}\}; 3 \in \{3, \{3\}\}$
(ii) $\{3\} \subseteq \{3, 3\}; \{3\} \not\subseteq \{\{\{3\}\}, \{3\}\}; \{3\} \not\subseteq \{\{\{\{3\}\}\}; \{3\} \subseteq \{3, \{3\}\}$
Note that $\{x\} \subseteq S$ if and only if $x \in S$.

2. • f is not one-to-one, because $f(3) = f(4)$
• f is not onto, because there is no x such that $f(x) = 4$
• $f \circ f$ is not a bijection, because $f(f(3)) = f(f(4))$ and also because there is no x such that $f(f(x)) = 4$.

3. (a)

$$T(i) = \frac{3}{2^{i-1}}$$

- (b)

$$\begin{aligned} S(n) &= \sum_{i=1}^n T(i) \\ &= \sum_{i=1}^n \frac{3}{2^{i-1}} \\ &= \sum_{i=0}^{n-1} \frac{3}{2^i} \\ S(n+1) &= \sum_{i=0}^n 3 \left(\frac{1}{2}\right)^i \\ &= \frac{3(1 - (\frac{1}{2})^{n+1})}{1 - \frac{1}{2}} \\ &= 6(1 - (\frac{1}{2})^{n+1}) \\ S(n) &= 6(1 - (\frac{1}{2})^n) \end{aligned}$$