RULES OF EQUIVALENCE for propositional logic: (Rosen same as lecture)

$\rightarrow$ equivalence	$p \rightarrow q \equiv \neg p \vee q$	
$\leftrightarrow$ equivalence	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	
2-negation	$\neg\neg q \equiv q$	
DeMorgan's Laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
Associative[a]	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Commutative[a]	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Distributive	$(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$	$(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$
Idempotence	$p \vee p \equiv p$	$p \wedge p \equiv p$
Inverse[b]	$p \vee \neg p \equiv True$	$p \wedge \neg p \equiv False$
Identity	$p \vee False \equiv p$	$p \wedge True \equiv p$
Domination	$p \wedge False \equiv False$	$p \vee True \equiv True$

[a] Often ommitted as trivial.
[b] Called "Negation" in edition 7.

Inferences rules of NATURAL DEDUCTION for propositional logic :

	Introduction	Elimination
AND	$\dfrac{p,q}{p \wedge q}$	$\dfrac{p \wedge q}{p}$ $\dfrac{p \wedge q}{q}$
OR	$\dfrac{p}{p \vee q}$ $\dfrac{q}{p \vee q}$	$p \vee q$, [from assumption p deduce r] [from assumption q deduce r] / r
$\leftrightarrow$	$\dfrac{p \longrightarrow q, q \longrightarrow p}{p \longleftrightarrow q}$	$\dfrac{p \longleftrightarrow q}{p \longrightarrow q}$ $\dfrac{p \longleftrightarrow q}{q \longrightarrow p}$
$\rightarrow$	[from assumption p you deduce q] / $p \longrightarrow q$	$\dfrac{p, \quad p \longrightarrow q}{q}$
NOT	[from assumption p you deduce False] / $\neg p$	$\dfrac{\neg\neg p}{p}$
False	$\dfrac{p \ , \ \neg p}{False}$	$\dfrac{False}{r}$

Basic additional inference rules for QUANTIFIERS:

All-ELIM (UI)	$\dfrac{\forall x.P(x)}{P(c)}$	where c is an **arbitrary** new constant, or one previously used
All-INTRO (UG)	$\dfrac{P(d)}{\forall x.P(x)}$	where d is an **arbitrary** constant
Exists-ELIM (EI)	$\exists x.P(x)$ [from assumption P(c) deduce B] / B	where e is a new constant, which no longer occurs in B
Exists-INTRO (EG)	$\dfrac{P(b)}{\exists y.P(y)}$	where b is any constant

Inference rules from Rosen's textbook

Table 2: Rules of inference for *Quantified Statements***:**

Name	*Rule of Inference*	
Universal instantiation (UI)	$\dfrac{\forall x.P(x)}{P(c)}$	(for any arbitrary c)
Universal generalization (UG)	$\dfrac{P(c)}{\forall x.P(x)}$	for an arbitrary c
Existential instantiation (UI)	$\dfrac{\exists x.P(x)}{P(e)}$	for some element e
Existential generalization (EG)	$\dfrac{P(e)}{\exists y.P(y)}$	for some element e

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution