Show all work clearly and in order, and circle your final answers. Justify your answers alge-braically whenever possible; when you do use your calculator, sketch all relevant graphs and write down all relevant mathematics. You have 30 minutes to take this 15 point quiz.

1. (*5 points*) Using **only** natural deduction prove the following statements;

1. $(p \rightarrow (q \rightarrow r)) \vdash (q \rightarrow (p \rightarrow r))$ (2.5 points)

2. $(p \rightarrow q) \vdash ((q \rightarrow r) \rightarrow (p \rightarrow r))$ (2.5 points)

$$\boxed{\text{Use backside for the solutions}}$$

2. (*2 points*) Briefly describe what is **proof by contradiction** and **proof by contrapositive** for a statement $p \rightarrow q$.

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3. (*8 points*) (a) Using proof by cases prove the following statement;

$$\text{If } n \text{ is an integer then } 3n^2 + n + 14. \text{ is even.}$$

(b) Using proof by contradiction show that the equation $x^2 - y^2 = 10$, has no positive integer solutions for x and y. (**Hint**: May be useful, $a^2 - b^2 = (a+b)(a-b)$.)

$$\boxed{\text{Use backside for the solutions}}$$